

\* Stefan's law :- The experimental study of the rate of emission of heat energy by a hot body Tyndall helped Stefan (in 1879) to enunciate the law called Stefan's law. In 1884, Boltzmann gave a theoretical proof of Stefan's law on the basis of thermodynamics. Therefore, this law is also called Stefan - Boltzmann law.

According to this law, the rate of emission of radiant energy by unit area of a perfectly black body is directly proportional to the fourth power of its absolute temperature.

$$R \propto T^4$$

$$\text{or, } R = \sigma T^4 \quad \text{--- (i)}$$

Where  $\sigma$  is called the Stefan's constant. If the body is not perfectly black and its emissivity or relative emittance is  $e$ , then

$$R = e \sigma T^4 \quad \text{--- (ii)}$$

Hence  $e$  varies between zero and one; depending on the nature of the surface. For a perfectly black body  $e=1$ . The law is not only true for emission but also for absorption of radiant energy.

Hence if a perfectly black body at temperature  $T_1$  is surrounded by a wall at a

temperature  $T_2$ , the net rate of loss of heat energy per unit area of the surface is given by

$$R \propto (T_1^4 - T_2^4)$$

$$R = \sigma (T_1^4 - T_2^4) \quad \text{--- (23)}$$

If the body has an emissivity  $e$ , then

$$R = e \sigma (T_1^4 - T_2^4) \quad \text{--- (24)}$$

\* Mathematical Derivation of Stefan's Law :-

The fact that black body radiations exert pressure similar to a gas, helps in applying thermodynamics to heat radiations.

Let  $\psi$  be the energy density of radiations inside a uniform temp. enclosure at temp  $T$ .  $P$  is the pressure and  $V$  is the volume.

Applying the first law of thermodynamics,

$$\delta H = dU + P \cdot dV \quad \text{--- (i)}$$

Applying thermodynamical relation,

$$\left( \frac{\partial H}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V \quad \text{--- (vi)}$$

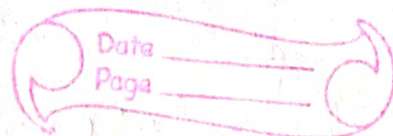
$$\left( \frac{\partial U + P \partial V}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V$$

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P \quad \text{--- (vii)}$$

Now  $U = V \psi$

and  $P = \frac{\psi}{3}$

$$\left[ \frac{\partial U}{\partial V} \right]_T = \psi$$



Here  $\psi$  is a function of temp<sup>r</sup>. alone.

Substituting these values in eqn. (vii), we get,

$$\psi = \frac{I}{3} \cdot \frac{d\psi}{dT} - \frac{\psi}{3}$$

$$\frac{4\psi}{3} = \frac{I}{3} \cdot \frac{d\psi}{dT}$$

$$\frac{d\psi}{\psi} = 4 \cdot \frac{dT}{T}$$

Integrating both sides,

$$\log \psi = 4 \log T + \text{constant}$$

$$\psi = kT^4 \quad \text{--- (viii)}$$

Here  $k$  is constant

Also, the total rate of emission per unit area of a black body is proportional to the energy density.

$$\therefore R \propto \psi \propto T^4$$

$$\therefore R = \sigma T^4 \quad \text{--- (ix)}$$

where  $\sigma$  is Stefan's constant.

The value of Stefan's constant in C.G.S. system is  $5.672 \times 10^{-5}$  C.G.S. unit, and in M.K.S. system, it is  $5.672 \times 10^{-8}$  M.K.S. units.